Class IX Chapter 1 – Number Sustems Maths

Exercise 1.1 Question

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0?

Answer:

Yes. Zero is a rational number as it can be represented as $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ etc.

Question 2:

Find six rational numbers between 3 and 4.

Answer:

There are infinite rational numbers in between 3 and 4.

3 and 4 can be represented as $\frac{24}{8}$ and $\frac{32}{8}$ respectively.

Therefore, rational numbers between 3 and 4 are

$\frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8}, \frac{29}{8}, \frac{30}{8}$

Question 3:

Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$. Answer:

There are infinite rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

$$\frac{3}{5}=\frac{3\times 6}{5\times 6}=\frac{18}{30}$$

$$\frac{4}{5}=\frac{4\times 6}{5\times 6}=\frac{24}{30}$$

Therefore, rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are

$$\frac{19}{30},\frac{20}{30},\frac{21}{30},\frac{22}{30},\frac{23}{30}$$

Question 4:

State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Answer:

(i) True; since the collection of whole numbers contains all natural numbers.

(ii) False; as integers may be negative but whole numbers are positive. For example: −3 is an integer but not a whole number.

(iii) False; as rational numbers may be fractional but whole numbers may not be. For example: $\frac{1}{5}$ is a rational number but not a whole number.

Exercise 1.2 Question 1:

State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Answer:

(i) True; since the collection of real numbers is made up of rational and irrational numbers.

(ii) False; as negative numbers cannot be expressed as the square root of any other number.

(iii) False; as real numbers include both rational and irrational numbers. Therefore, every real number cannot be an irrational number.

Question 2:

Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Answer:

If numbers such as $\sqrt{4} = 2, \ \sqrt{9} = 3$ are considered, Then here, 2 and 3 are rational numbers. Thus, the square roots of all positive integers are not irrational.

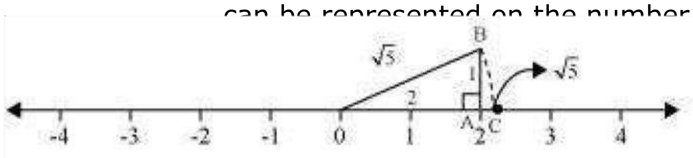
Question 3:

$\sqrt{5}$

Answer:

We know that, $\sqrt{4} = 2$

Show howAnd, $\sqrt{5} = \sqrt{(2)^2 + (1)^2}$ can be represented on the number line.



Mark a point ‘A’ representing 2 on number line. Now, construct AB of unit length perpendicular to OA. Then, taking O as centre and OB as radius, draw an arc intersecting number line at C.

C is representing $\sqrt{5}$.

has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Answer:

(i) $\frac{36}{100} = 0.36$

Terminating

(ii) $\frac{1}{11} = 0.090909......$ $= 0.\overline{09}$

Non-terminating repeating

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$

Terminating

(iv) $\frac{3}{13} = 0.230769230769....$ $= 0.\overline{230769}$

Non-terminating repeating

(v) $\frac{2}{11} = 0.18181818........$ $= 0.\overline{18}$

Non-terminating repeating

(vi) $\frac{329}{400} = 0.8225$

Terminating

$\frac{1}{7} = 0.\overline{142857}$ Question 2:

You know that

$\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$

Exercise 1.3 Question 1:

Write the following in decimal form and say what kind of decimal expansion each . Can you predict what the decimal expansion of

are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.] Answer:

Yes. It can be done as follows.

$$\frac{2}{7} = 2\times\frac{1}{7} = 2\times 0.\overline{142857} = 0.\overline{285714}$$

$$\frac{3}{7} = 3\times\frac{1}{7} = 3\times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4\times\frac{1}{7} = 4\times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5\times\frac{1}{7} = 5\times 0.\overline{142857} = 0.\overline{714285}$$

$$\frac{6}{7} = 6\times\frac{1}{7} = 6\times 0.\overline{142857} = 0.\overline{857142}$$

Question 3:

Express the following in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Answer:

(i) $0.\overline{6} = 0.666...$

Let x = 0.666...

10x = 6.666...

10x = 6 + x

9x = 6

$x = \frac{2}{3}$

(ii) $0.4\overline{7} = 0.4777.....$

$= \frac{4}{10} + \frac{0.777}{10}$

Let x = 0.777...

10x = 7.777...

10x = 7 + x

$x = \frac{7}{9}$

$$\frac{4}{10} + \frac{0.777...}{10} = \frac{4}{10} + \frac{7}{90}$$
$$= \frac{36+7}{90} = \frac{43}{90}$$

(iii) $0.\overline{001} = 0.001001...$

Let x = 0.001001...

1000x = 1.001001...

1000x = 1 + x

999x = 1

$x = \frac{1}{999}$

Question 4:

Express 0.99999...in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer:

Let x = 0.9999...

10x = 9.9999…

10x = 9 + x

9x = 9 x =

1

Question 5:

What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Answer:

It can be observed that,

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$.

Question 6:

Look at several examples of rational numbers in the form $\frac{p}{q}$ (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer:

Terminating decimal expansion will occur when denominator q of rational number $\frac{p}{q}$ is either of 2, 4, 5, 8, 10, and so on…

$$\frac{9}{4} = 2.25$$

$$\frac{11}{8} = 1.375$$

$$\frac{27}{5} = 5.4$$

It can be observed that terminating decimal may be obtained in the situation where prime factorisation of the denominator of the given fractions has the power of 2 only or 5 only or both.

Question 7:

Write three numbers whose decimal expansions are non-terminating non-recurring.
Answer:

3 numbers whose decimal expansions are non-terminating non-recurring are as follows.

0.505005000500005000005...

0.7207200720007200007200000... 0.08008000800008000008000008...

Question

8:

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$. Answer:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

3 irrational numbers are as follows.

0.73073007300073000073...

0.75075007500075000075... 0.79079007900079000079...

Question

9:

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 (v) 1.101001000100001...

(i) $\sqrt{23} = 4.79583152331\ ...$

As the decimal expansion of this number is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $\sqrt{225} = 15 = \frac{15}{1}$

It is a rational number as it can be represented in $\frac{p}{q}$ form.

(iii) 0.3796

As the decimal expansion of this number is terminating, therefore, it is a rational number.

(iv) 7.478478 … $= 7.\overline{478}$

As the decimal expansion of this number is non-terminating recurring, therefore, it is a rational number.

(v) 1.10100100010000 …

As the decimal expansion of this number is non-terminating non-repeating, therefore, it is an irrational number.

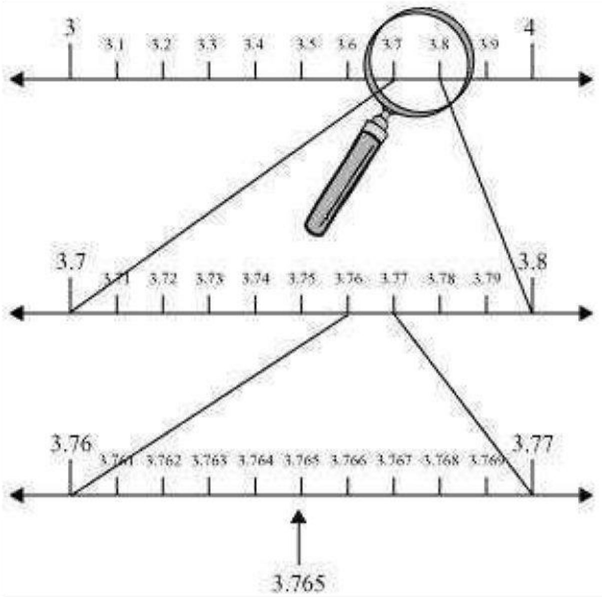
Exercise 1.4 Question

1:

Visualise 3.765 on the number line using successive magnification.

Answer:

3.765 can be visualised as in the following steps.



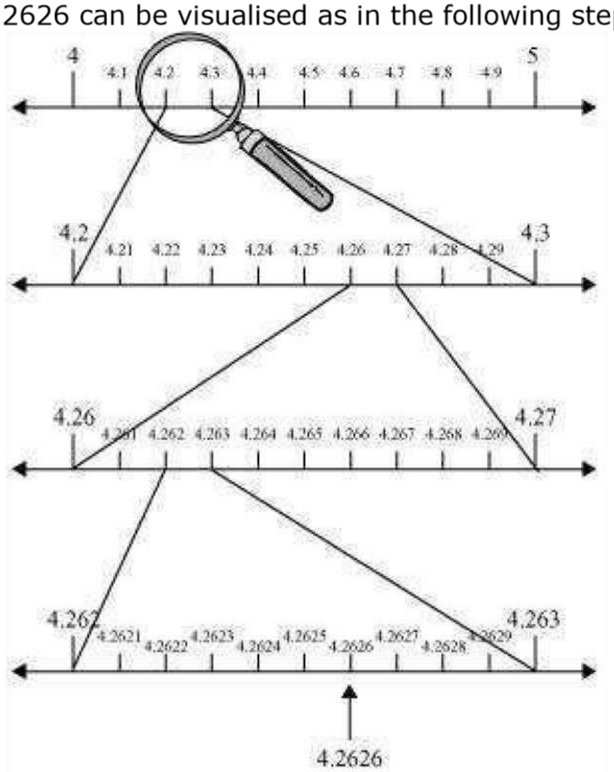
Question 2:

Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Answer:

$4.\overline{26}$ = 4.2626…

4.2626 can be visualised as in the following steps.



Exercise 1.5 Question 1:

1Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$ (v) 2π

Answer:

(i) $2-\sqrt{5}$ = 2 − 2.2360679…

= − 0.2360679…

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

form, therefore, it is a rational

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}=3=\dfrac{3}{1}$

number. form, therefore, it is a

As it can be represented in $\dfrac{p}{q}$

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}=\dfrac{2}{7}$

As it can be represented in $\dfrac{p}{q}$

rational number.

(iv) $\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}=0.7071067811...$

As the decimal expansion of this expression is non-terminating non-recurring,

therefore, it is an irrational number. (v)
$2\pi = 2(3.1415 \ldots)$

$= 6.2830 \ldots$
As the decimal expansion of this expression is non-terminating non-recurring, therefore,

it is an irrational number.

Question 2:

Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$ (iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Answer:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$

$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=(3)^2-\left(\sqrt{3}\right)^2$

$= 9 - 3 = 6$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{2}\right)$

$=5+2+2\sqrt{10}=7+2\sqrt{10}$

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$

$= 5 - 2 = 3$

Question 3:

Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi=\frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer:

There is no contradiction. When we measure a length with scale or any other instrument, we only obtain an approximate rational value. We never obtain an exact value. For this reason, we may not realise that either c or d is irrational. Therefore,

the $\frac{c}{d}$ fraction is irrational. Hence, π is irrational.

Question $\sqrt{9.3}$ 4:

Represent on the number line.

Answer:

Mark a line segment OB = 9.3 on number line. Further, take BC of 1 unit. Find the midpoint

D of OC and draw a semi-circle on OC while taking D as its centre. Draw a

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

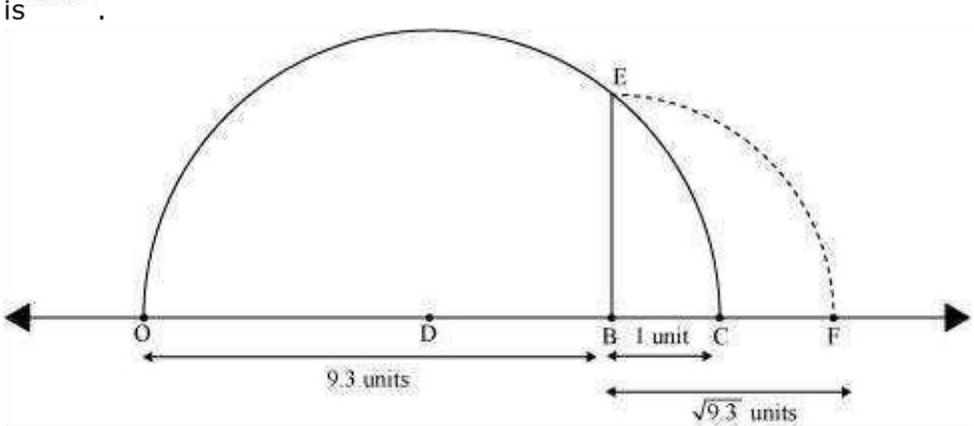
(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Answer:

(i) $\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{1\times\sqrt{7}} = \frac{\sqrt{7}}{7}$

perpendicular to line OC passing through point B. Let it intersect the semi-circle at E.

Taking B as centre and BE as radius, draw an arc in tersecting number line at F. BF is $\sqrt{9.3}$.



Question 5:

Rationalise the denominators of the following:

(ii) $$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\left(\sqrt{7}-\sqrt{6}\right)}\frac{\left(\sqrt{7}+\sqrt{6}\right)}{\left(\sqrt{7}+\sqrt{6}\right)}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\frac{\sqrt{7}+\sqrt{6}}{1}=\sqrt{7}+\sqrt{6}$$

(iii) $$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{1}{\left(\sqrt{5}+\sqrt{2}\right)}\frac{\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}-\sqrt{2}\right)}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv) $$\frac{1}{\sqrt{7}-2}=\frac{1}{\left(\sqrt{7}-2\right)}\frac{\left(\sqrt{7}+2\right)}{\left(\sqrt{7}+2\right)}$$

$$=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}=\frac{\sqrt{7}+2}{3}$$

Exercise 1.6 Question 1:

Find:

(i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Find:

(i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Answer:
Answer:

(i)

$$64^{\frac{1}{2}} = \left(2^6\right)^{\frac{1}{2}}$$
$$= 2^{6\times\frac{1}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^3 = 8$$

(ii)

$$32^{\frac{1}{5}} = \left(2^5\right)^{\frac{1}{5}}$$
$$= (2)^{5\times\frac{1}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^1 = 2$$

(iii)

$$(125)^{\frac{1}{3}} = \left(5^3\right)^{\frac{1}{3}}$$
$$= 5^{3\times\frac{1}{3}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 5^1 = 5$$

Question 2:

(i)

$$9^{\frac{3}{2}} = \left(3^2\right)^{\frac{3}{2}}$$
$$= 3^{2\times\frac{3}{2}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 3^3 = 27$$

(ii)

$$(32)^{\frac{2}{5}} = \left(2^5\right)^{\frac{2}{5}}$$
$$= 2^{5\times\frac{2}{5}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^2 = 4$$

(iii)

$$(16)^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}}$$
$$= 2^{4\times\frac{3}{4}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= 2^3 = 8$$

(iv)

$$(125)^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}} \qquad \left[a^{-m} = \frac{1}{a^m}\right]$$
$$= \frac{1}{\left(5^3\right)^{\frac{1}{3}}}$$
$$= \frac{1}{5^{3\times\frac{1}{3}}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$
$$= \frac{1}{5}$$

Question 3:

Simplify:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Answer:

(i)

$$2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} \qquad \left[a^m.a^n = a^{m+n}\right]$$

$$= 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$$

(ii)

$$\left(\frac{1}{3^3}\right)^7 = \frac{1}{3^{3\times7}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= \frac{1}{3^{21}}$$

$$= 3^{-21} \qquad \left[\frac{1}{a^m} = a^{-m}\right]$$

(iii)

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right]$$

$$= 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$$

(iv)

$$7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7\times8)^{\frac{1}{2}} \qquad [a^m.b^m = (ab)^m]$$

$$= (56)^{\frac{1}{2}}$$

Class IX Chapter 2 Polynomials Maths

Exercise 2.1 Question

1:

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$ (v) $x^{10} + y^3 + t^{50}$

Answer:

(i) $4x^2 - 3x + 7$

Yes, this expression is a polynomial in one variable x.

(ii) $y^2 + \sqrt{2}$

Yes, this expression is a polynomial in one variable y.

(iii) $3\sqrt{t} + t\sqrt{2}$

No. It can be observed that the exponent of variable t in term $3\sqrt{t}$ is $\frac{1}{2}$, which is not a whole number. Therefore, this expression is not a polynomial.

(iv) $y + \frac{2}{y}$

No. It can be observed that the exponent of variable y in term $\frac{2}{y}$ is −1, which is not a whole number. Therefore, this expression is not a polynomial.

(v) $x^{10}+y^3+t^{50}$

No. It can be observed that this expression is a polynomial in 3 variables x, y, and t.

Therefore, it is not a polynomial in one variable.

Question 2:

Write the coefficients of x^2 in each of the following:
(i) $2+x^2+x$ (ii) $2-x^2+x^3$
(iii) $\frac{\pi}{2}x^2+x$ (iv) $\sqrt{2}x-1$
Answer:
(i) $2+x^2+x$

In the above expression, the coefficient of x^2 is 1.

(ii) $2-x^2+x^3$

In the above expression, the coefficient of x^2 is −1.

(iii) $\frac{\pi}{2}x^2+x$

In the above expression, the coefficient of x^2 is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1$, or
$0.x^2+\sqrt{2}x-1$

In the above expression, the coefficient of x^2 is 0.

Question 3:

Give one example each of a binomial of degree 35, and of a monomial of degree 100.
Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

Binomial has two terms in it. Therefore, binomial of degree 35 can be written as

$x^{35} + x^{34}$.

Monomial has only one term in it. Therefore, monomial of degree 100 can be written as x^{100}.

Question 4:

Write the degree of each of the following polynomials:

$5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$ (i)

$5t - \sqrt{7}$ (iv) 3 (iii)

Answer:

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) $5x^3 + 4x^2 + 7x$

This is a polynomial in variable x and the highest power of variable x is 3. Therefore, the degree of this polynomial is 3.

(ii) $4 - y^2$

This is a polynomial in variable y and the highest power of variable y is 2. Therefore, the degree of this polynomial is 2.

(iii) $5t - \sqrt{7}$

This is a polynomial in variable t and the highest power of variable t is 1. Therefore, the degree of this polynomial is 1.

(iv) 3

This is a constant polynomial. Degree of a constant polynomial is always 0.

Question 5:

Classify the following as linear, quadratic and cubic polynomial:

$x^2 + x$ (ii) $x - x^3$ (iii) $y + y^2 + 4$ (iv) $1 + x$ (v) $3t$ (i)

r^2 (vii) $7x^3$

(vi)

Answer:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively.

$x^2 + x$

(i) is a quadratic polynomial

$x - x^3$ as its degree is 2.

$y + y^2 + 4$

(ii) is a cubic polynomial as its degree is 3. (iii) is a quadratic polynomial as its degree is 2.

$3t$

r^2 (iv) 1 + x is a linear polynomial as its degree is 1.

$7x^3$

(v) is a linear polynomial as its degree is 1.

(vi) is a quadratic polynomial as its degree is 2.

(vii) is a cubic polynomial as its degree is 3.

Exercise 2.2 Question

1:

Find the value of the polynomial $5x-4x^2+3$ at

(i) x = 0 (ii) x = −1 (iii) x = 2

Answer:

(i) $p(x)=5x-4x^2+3$

$$p(0)=5(0)-4(0)^2+3$$
$$=3$$

(ii) $p(x)=5x-4x^2+3$

$$p(-1)=5(-1)-4(-1)^2+3$$
$$=-5-4(1)+3=-6$$

(iii) $p(x)=5x-4x^2+3$

$$p(2)=5(2)-4(2)^2+3$$
$$=10-16+3=-3$$

Question 2:

Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

Answer: (i) $p(y) = y^2 - y + 1$ $p(0) =$

$(0)^2 - (0) + 1 = 1$ $p(1) = (1)^2 - (1) + 1 =$

1 $p(2) = (2)^2 - (2) + 1 = 3$ (ii) $p(t) =$

$2 + t + 2t^2 - t^3$ $p(0) = 2 + 0 + 2(0)^2 -$
$(0)^3 = 2$ $p(1) = 2 + (1) + 2(1)^2 - (1)^3$

$= 2 + 1 + 2 - 1 = 4$ $p(2) =$

$2 + 2 + 2(2)^2 - (2)^3$

$= 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$ $p(0) = (0)^3 = 0$ $p(1) =$

$(1)^3 = 1$ $p(2) = (2)^3 = 8$ (iv) $p(x) = (x - 1)(x$

$+ 1)$ $p(0) = (0 - 1)(0 + 1) = (-1)$

$(1) = -1$ $p(1) = (1 - 1)(1 + 1) = 0$ $(2) = 0$
$p(2) = (2 - 1)(2 + 1) = 1(3) = 3$ Question
3:

Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, x = -\frac{1}{3}$ (ii) $p(x) = 5x - \pi, x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1$, $x = 1, -1$ (iv) $p(x) = (x + 1)(x - 2)$, $x = -1, 2$

(v) $p(x) = x^2$, $x = 0$ (vi) $p(x) = lm + m, x = -\frac{m}{l}$

(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ (viii) $p(x) = 2x + 1, x = \frac{1}{2}$

Answer:

(i) If $x = \frac{-1}{3}$ is a zero of given polynomial $p(x) = 3x + 1$, then $p\left(-\frac{1}{3}\right)$ should be 0.

$$\text{Here, } p\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right) + 1 = -1 + 1 = 0$$

Therefore, $x = \frac{-1}{3}$ is a zero of the given polynomial.

(ii) If $x = \frac{4}{5}$ is a zero of polynomial $p(x) = 5x - \pi$, then $p\left(\frac{4}{5}\right)$ should be 0.

$$\text{Here, } p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi$$

$$\text{As } p\left(\frac{4}{5}\right) \neq 0,$$

Therefore, $x = \frac{4}{5}$ is not a zero of the given polynomial.

(iii) If $x = 1$ and $x = -1$ are zeroes of polynomial $p(x) = x^2 - 1$, then $p(1)$ and $p(-1)$ should be 0.

Here, $p(1) = (1)^2 - 1 = 0$, and $p(-1)$

$= (-1)^2 - 1 = 0$

Hence, $x = 1$ and -1 are zeroes of the given polynomial.

(iv) If $x = -1$ and $x = 2$ are zeroes of polynomial $p(x) = (x + 1)(x - 2)$, then $p(-1)$ and $p(2)$ should be 0.

Here, $p(-1) = (-1 + 1)(-1 - 2) = 0(-3) = 0$, and $p(2)$

$= (2 + 1)(2 - 2) = 3(0) = 0$

Therefore, $x = -1$ and $x = 2$ are zeroes of the given polynomial.

(v) If $x = 0$ is a zero of polynomial $p(x) = x^2$, then $p(0)$ should be zero.

Here, $p(0) = (0)^2 = 0$

Hence, $x = 0$ is a zero of the given polynomial.

(vi) If $x = \frac{-m}{l}$ is a zero of polynomial $p(x) = lx + m$, then should be 0.

Here, $p\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m = -m + m = 0$

Therefore, $x = -\frac{m}{l}$ is a zero of the given polynomial.

(vii) If $x = \frac{-1}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ are zeroes of polynomial $p(x) = 3x^2 - 1$, then

$p\left(\frac{-m}{l}\right)$

$p\left(\frac{-1}{\sqrt{3}}\right)$ and $p\left(\frac{2}{\sqrt{3}}\right)$ should be 0.

Here, $p\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$, and

$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$

Hence, $x = \frac{-1}{\sqrt{3}}$ is a zero of the given polynomial. However, $x = \frac{2}{\sqrt{3}}$ is not a zero of the given polynomial.

(viii) If $x = \frac{1}{2}$ is a zero of polynomial p(x) = 2x + 1, then $p\left(\frac{1}{2}\right)$ should be 0.

Here, $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

As $p\left(\frac{1}{2}\right) \neq 0$,

Therefore, $x = \frac{1}{2}$ is not a zero of the given polynomial.

Question 4:

Find the zero of the polynomial in each of the following cases:
(i) p(x) = x + 5 (ii) p(x) = x − 5 (iii) p(x) = 2x + 5 (iv) p(x)

= 3x − 2 (v) p(x) = 3x (vi) p(x) = ax, a ≠ 0 (vii) p(x) = cx + d,
c ≠ 0, c, are real numbers.

Answer:

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.

(i) p(x) = x + 5 p(x)

= 0 x + 5 = 0 x = − 5

Therefore, for x = −5, the value of the polynomial is 0 and hence, x = −5 is a zero of the given polynomial. (ii)

p(x) = x − 5
p(x) = 0 x − 5

= 0 x = 5

Therefore, for x = 5, the value of the polynomial is0 and hence, x = 5 is a zero of the given polynomial. (iii) p(x) = 2x + 5 p(x) = 0

2x + 5 = 0

2x = − 5

$x = -\frac{5}{2}$

Therefore, for $x = -\frac{5}{2}$, the value of the polynomial is 0 and hence, $x = \frac{-5}{2}$ is a zero of the given polynomial. (iv) p(x) = 3x − 2 p(x) = 0

3x − 2 = 0

$x = \frac{2}{3}$

Therefore, for $x = \frac{2}{3}$, the value of the polynomial is 0 and hence, $x = \frac{2}{3}$ is a zero of the given polynomial. (v) p(x) = 3x p(x) = 0 3x = 0 x = 0

Therefore, for x = 0, the value of the polynomial is 0 and hence, x = 0 is a zero of the given polynomial. (vi) p(x) = ax p(x) = 0 ax = 0 x = 0

Therefore, for x = 0, the value of the polynomial is 0 and hence, x = 0 is a zero of the

given polynomial. (vii) p(x) = cx + d p(x) = 0 cx+ d = 0

$x = \frac{-d}{c}$

Therefore, for $x = \frac{-d}{c}$, the value of the polynomial is 0 and hence, $x = \frac{-d}{c}$ is a zero of the

given polynomial.

Exercise 2.3 Question

1:

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$ (ii) $x-\frac{1}{2}$ (iii) x

(iv) $x + \pi$ (v) $5 + 2x$ Answer:

(i) x + 1

By long division,

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
x+1 & x^3 + 3x^2 + 3x + 1 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & \quad 2x^2 + 3x + 1 \\
 & \quad 2x^2 + 2x \\
 & \quad - \quad - \\
\hline
 & \qquad x + 1 \\
 & \qquad x + 1 \\
 & \qquad - \quad - \\
\hline
 & \qquad 0 \\
\hline
\end{array}
$$

Therefore, the remainder is 0.

(ii) $x - \frac{1}{2}$

By long division,

$$
\begin{array}{r|l}
 & x^2 + \frac{7}{2}x + \frac{19}{4} \\
\cline{2-2}
x - \frac{1}{2} & x^3 + 3x^2 + 3x + 1 \\
 & x^3 - \frac{x^2}{2} \\
 & - \quad + \\
\cline{2-2}
 & \frac{7}{2}x^2 + 3x + 1 \\
 & \frac{7}{2}x^2 - \frac{7}{4}x \\
 & - \quad + \\
\cline{2-2}
 & \frac{19}{4}x + 1 \\
 & \frac{19}{4}x - \frac{19}{8} \\
 & - \quad + \\
\cline{2-2}
 & \frac{27}{8} \\
\cline{2-2}
\end{array}
$$

Therefore, the remainder is $\frac{27}{8}$.

(iii) x

By long division,

$$
\begin{array}{r|l}
 & x^2 + 3x + 3 \\
\cline{2-2}
x & x^3 + 3x^2 + 3x + 1 \\
 & x^3 \\
 & - \\
\cline{2-2}
 & 3x^2 + 3x + 1 \\
 & 3x^2 \\
 & - \\
\cline{2-2}
 & 3x + 1 \\
 & 3x \\
 & - \\
\cline{2-2}
 & 1 \\
\cline{2-2}
\end{array}
$$

Therefore, the remainder is 1.

(iv) x + π

By long division,

$$
\begin{array}{r|l}
 & x^2+(3-\pi)x+\left(3-3\pi+\pi^2\right) \\
\hline
x+\pi & x^3+3x^2+3x+1 \\
 & x^3+\pi x^2 \\
 & - \quad - \\
\hline
 & (3-\pi)x^2+3x+1 \\
 & (3-\pi)x^2+(3-\pi)\pi x \\
 & - \qquad - \\
\hline
 & \left[3-3\pi+\pi^2\right]x+1 \\
 & \left[3-3\pi+\pi^2\right]x+\left(3-3\pi+\pi^2\right)\pi \\
 & - \qquad\qquad - \\
\hline
 & \left[1-3\pi+3\pi^2-\pi^3\right] \\
\hline
\end{array}
$$

Therefore, the remainder is $-\pi^3+3\pi^2-3\pi+1.$

(v) $5 + 2x$

By long division,

$$
\begin{array}{r l}
 & \frac{x^2}{2} + \frac{x}{4} + \frac{7}{8} \\
2x+5 \,\big) & x^3 + 3x^2 + 3x + 1 \\
 & x^3 + \frac{5}{2}x^2 \\
 & - \quad - \\
\hline
 & \frac{x^2}{2} + 3x + 1 \\
 & \frac{x^2}{2} + \frac{5x}{4} \\
 & - \quad - \\
\hline
 & \frac{7x}{4} + 1 \\
 & \frac{7}{4}x + \frac{35}{8} \\
 & - \quad - \\
\hline
 & -\frac{27}{8} \\
\hline
\end{array}
$$

Therefore, the remainder is $-\frac{27}{8}$

Question 2:

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Answer:

By long division,

$$
\begin{array}{r l}
 & x^2 + 6 \\
x-a \,\big) & x^3 - ax^2 + 6x - a \\
 & x^3 - ax^2 \\
 & - \quad + \\
\hline
 & 6x - a \\
 & 6x - 6a \\
 & - \quad + \\
\hline
 & 5a \\
\hline
\end{array}
$$

Therefore, when $x^3 - ax^2 + 6x - a$ is divided by $x - a$, the remainder obtained is $5a$.

Question 3:

Check whether 7 + 3x is a factor of $3x^3 + 7x$.

Answer:

Let us divide ($3x^3 + 7x$) by (7 + 3x). If the remainder obtained is 0, then 7 + 3x will be a factor of $3x^3 + 7x$.

By long division,

$$
\begin{array}{r|l}
 & x^2 - \frac{7}{3}x + \frac{70}{9} \\
\hline
3x+7 & 3x^3 + 0x^2 + 7x \\
 & 3x^3 + 7x^2 \\
 & - \quad - \\
\hline
 & -7x^2 + 7x \\
 & -7x^2 - \frac{49x}{3} \\
 & + \quad + \\
\hline
 & \frac{70x}{3} \\
 & \frac{70x}{3} + \frac{490}{9} \\
 & - \quad - \\
\hline
 & -\frac{490}{9} \\
\hline
\end{array}
$$

As the remainder is not zero, therefore, 7 + 3x is not a factor of $3x^3 + 7x$.

Exercise 2.4

Question 1:

Determine which of the following polynomials has (x + 1) a factor:

(i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ (iv) $x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$

Answer:

(i) If (x + 1) is a factor of $p(x) = x^3 + x^2 + x + 1$, then p (−1) must be zero, otherwise (x + 1) is not a factor of p(x).

$p(x) = x^3 + x^2 + x + 1$ $p(-1) =$

$(-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 - 1 = 0$

Hence, x + 1 is a factor of this polynomial.

(ii) If (x + 1) is a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$, then p (−1) must be zero, otherwise (x + 1) is not a factor of p(x). $p(x) = x^4 + x^3 + x^2 + x + 1$ $p(-1) =$

$(-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$
$= 1 - 1 + 1 - 1 + 1 = 1$

As p $\neq 0$, (− 1)

Therefore, x + 1 is not a factor of this polynomial.

(iii) If (x + 1) is a factor of polynomial $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$, then p(−1) must be 0, otherwise (x + 1) is not a factor of this polynomial.

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

As $p \neq 0, (-1)$

Therefore, $x + 1$ is not a factor of this polynomial.

(iv) If$(x + 1)$ is a factor of polynomial $p(x) = x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$, then $p(-1)$ must be 0, otherwise $(x + 1)$ is not a factor of this polynomial.

$$\begin{aligned} p(-1) &= (-1)^3 - (-1)^2 - \left(2+\sqrt{2}\right)(-1) + \sqrt{2} \\ &= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

As $p \neq 0, (-1)$

Therefore, $(x + 1)$ is not a factor of this polynomial.

Question 2:

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$ (iii) $p(x)$ $= x^3 - 4x^2 + x + 6$, $g(x) = x - 3$ Answer:

(i) If $g(x) = x + 1$ is a factor of the given polynomial $p(x)$, then $p(-1)$ must be zero. $p(x) = 2x^3 + x^2 - 2x - 1$ $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= 2(-1) + 1 + 2 - 1 = 0$

Hence, $g(x) = x + 1$ is a factor of the given polynomial.

(ii) If $g(x) = x + 2$ is a factor of the given polynomial $p(x)$, then $p(-2)$ must be

0.

$p(x) = x^3 + 3x^2 + 3x + 1$ $p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -1$

As $p \neq 0, (-2)$

Hence, g(x) = x + 2 is not a factor of the given polynomial.

(iii) If g(x) = x − 3 is a factor of the given polynomial p(x), then p(3) must be 0.

$p(x) = x^3 - 4x^2 + x + 6$ p(3)

$= (3)^3 - 4(3)^2 + 3 + 6 = 27$

$- 36 + 9 = 0$

Hence, g(x) = x − 3 is a factor of the given polynomial.

Question 3:

Find the value of k, if x − 1 is a factor of p(x) in each of the following cases:

(i) $p(x) = x^2 + x + k$ (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ (iv) $p(x) = kx^2 - 3x + k$

Answer:

If x − 1 is a factor of polynomial p(x), then p(1) must be 0.

(i) $p(x) = x^2 + x + k$ p(1)

= 0

$\Rightarrow (1)^2 + 1 + k = 0$

$\Rightarrow (2) + k = 0 \Rightarrow k$

$= -2$

Therefore, the value of k is −2.

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

p(1) = 0

$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$

$\Rightarrow 2 + k + \sqrt{2} = 0$

$\Rightarrow k = -2 - \sqrt{2} = -(2 + \sqrt{2})$

Therefore, the value of k is $-(2+\sqrt{2})$.

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

p(1) = 0

$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k - \sqrt{2} + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$

Therefore, the value of k is $\sqrt{2}-1$.

(iv) $p(x) = kx^2 - 3x + k$

$\Rightarrow p(1) = 0 \Rightarrow k(1)^2 -$

$3(1) + k = 0 \Rightarrow k - 3$

$+ k = 0$

$\Rightarrow \quad 2k - 3 = 0$

$\Rightarrow k = \frac{3}{2}$

Therefore, the value of k is $\frac{3}{2}$.

Question 4:

Factorise:

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$ Answer: (i) $12x^2 - 7x + 1$

We can find two numbers such that pq = 12 × 1 = 12 and p + q = −7. They are p = −4 and q = −3.

Here, $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1)$
(ii) $2x^2 + 7x + 3$

We can find two numbers such that $pq = 2 \times 3 = 6$ and $p + q = 7$.

They are $p = 6$ and $q = 1$.

Here, $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3) = (x + 3)(2x + 1)$

(iii) $6x^2 + 5x - 6$

We can find two numbers such that $pq = -36$ and $p + q = 5$.

They are $p = 9$ and $q = -4$.

Here,

$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

(iv) $3x^2 - x - 4$

We can find two numbers such that $pq = 3 \times (-4) = -12$ and $p + q = -1$.

They are $p = -4$ and $q = 3$.

Here,

$3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$ Question 5:

Factorise:

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 + 3x^2 - 9x - 5$ (iii) $x^3 + 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$ Answer:

(i) Let $p(x) = x^3 - 2x^2 - x + 2$

All the factors of 2 have to be considered. These are $\pm 1, \pm 2$.

By trial method, $p(2) = (2)^3 - 2(2)^2 - 2 + 2$

$= 8 - 8 - 2 + 2 = 0$

Therefore, $(x - 2)$ is factor of polynomial $p(x)$.

Let us find the quotient on dividing $x^3 - 2x^2 - x + 2$ by $x - 2$.

By long division,

$$\begin{array}{r}
x^2 - 3x + 2 \\
x+1 \overline{\smash{\big)}\, x^3 - 2x^2 - x + 2} \\
x^3 + x^2 \qquad\qquad \\
- \quad - \qquad\qquad \\
\hline
-3x^2 - x + 2 \\
-3x^2 - 3x \quad\; \\
+ \quad + \quad\; \\
\hline
2x + 2 \\
2x + 2 \\
- \quad - \\
\hline
0 \\
\hline
\end{array}$$

It is known that,

Dividend = Divisor × Quotient + Remainder $\therefore x^3$

$- 2x^2 - x + 2 = (x + 1)(x^2 - 3x + 2) + 0 =$

$(x + 1)[x^2 - 2x - x + 2]$

$= (x + 1)[x(x - 2) - 1(x - 2)]$

$= (x + 1)(x - 1)(x - 2)$

$= (x - 2)(x - 1)(x + 1)$

(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$

All the factors of 5 have to be considered. These are ± 1, ± 5.

By trial method, $p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5 = 0$

Therefore, $x + 1$ is a factor of this polynomial.

Let us find the quotient on dividing $x^3 + 3x^2 - 9x - 5$ by $x + 1$.

By long division,

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
\cline{2-2}
x+1 & x^3 - 3x^2 - 9x - 5 \\
 & x^3 + x^2 \\
 & - \quad - \\
\cline{2-2}
 & -4x^2 - 9x - 5 \\
 & -4x^2 - 4x \\
 & + \quad + \\
\cline{2-2}
 & -5x - 5 \\
 & -5x - 5 \\
 & + \quad + \\
\cline{2-2}
 & 0 \\
\cline{2-2}
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder $\therefore x^3$

$- 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5) + 0$

$= (x + 1)(x^2 - 5x + x - 5)$

$= (x + 1)[(x(x - 5) + 1(x - 5)]$

$= (x + 1)(x - 5)(x + 1)$

$= (x - 5)(x + 1)(x + 1)$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

All the factors of 20 have to be considered. Some of them are ± 1,

$\pm 2, \pm 4, \pm 5$ …… By trial method, $p(-1)$

$= (-1)^3 + 13(-1)^2 +$

$32(-1) + 20$

$= - 1 + 13 - 32 + 20$

$= 33 - 33 = 0$

As p(−1) is zero, therefore, x + 1 is a factor of this polynomial p(x).

Let us find the quotient on dividing $x^3 + 13x^2 + 32x + 20$ by (x + 1).

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & \quad 12x^2 + 32x \\
 & \quad 12x^2 + 12x \\
 & \quad - \quad - \\
\hline
 & \qquad 20x + 20 \\
 & \qquad 20x + 20 \\
 & \qquad - \quad - \\
\hline
 & \qquad\quad 0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder x^3 +

$13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20) + 0$

$= (x + 1)(x^2 + 10x + 2x + 20)$

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 10)(x + 2) =$

$(x + 1)(x + 2)(x + 10)$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$

By trial method, $p(1) = 2(1)^3 +$

$(1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1 = 0$

Therefore, $y - 1$ is a factor of this polynomial.
Let us find the quotient on dividing $2y^3 + y^2 - 2y - 1$ by $y - 1$.

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & \quad 3y^2 - 2y - 1 \\
 & \quad 3y^2 - 3y \\
 & \quad - \quad + \\
\hline
 & \qquad y - 1 \\
 & \qquad y - 1 \\
\hline
 & \qquad 0 \\
\hline
\end{array}
$$

$p(y) = 2y^3 + y^2 - 2y - 1 =$

$(y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)[2y(y + 1) + 1(y + 1)]$

$= (y - 1)(y + 1)(2y + 1)$ Question
5:

Factorise:

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 + 3x^2 - 9x - 5$ (iii) x^3

$+ 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$

Answer:

(i) Let $p(x) = x^3 - 2x^2 - x + 2$

All the factors of 2 have to be considered. These are $\pm 1, \pm 2$.
By trial method, $p(2) = (2)^3 - 2(2)^2 - 2 + 2$

= 8 − 8 − 2 + 2 = 0

Therefore, (x − 2) is factor of polynomial p(x).

Let us find the quotient on dividing $x^3 - 2x^2 - x + 2$ by x − 2.

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x+1 & x^3 - 2x^2 - x + 2 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -3x^2 - x + 2 \\
 & -3x^2 - 3x \\
 & + \quad + \\
\hline
 & 2x + 2 \\
 & 2x + 2 \\
 & - \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder ∴

$x^3 - 2x^2 - x + 2 = (x + 1)(x^2 - 3x + 2) + 0$

$= (x + 1)[x^2 - 2x - x + 2]$

$= (x + 1)[x(x - 2) - 1(x - 2)]$

$= (x + 1)(x - 1)(x - 2)$

$= (x - 2)(x - 1)(x + 1)$

(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$

All the factors of 5 have to be considered. These are ±1, ± 5.
By trial method, $p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5 = 0$

Therefore, x + 1 is a factor of this polynomial.

Let us find the quotient on dividing $x^3 + 3x^2 - 9x - 5$ by x + 1.

By long division,

$$
\begin{array}{rl}
 & x^2 - 4x - 5 \\
x+1 \big) & x^3 - 3x^2 - 9x - 5 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -4x^2 - 9x - 5 \\
 & -4x^2 - 4x \\
 & + \quad + \\
\hline
 & -5x - 5 \\
 & -5x - 5 \\
 & + \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder $\therefore$ x^3

$- 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5) + 0$

$= (x + 1)(x^2 - 5x + x - 5)$

$= (x + 1)[(x(x - 5) + 1(x - 5)]$

$= (x + 1)(x - 5)(x + 1)$

$= (x - 5)(x + 1)(x + 1)$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

All the factors of 20 have to be considered. Some of them are ±1,

± 2, ± 4, ± 5 …… By

trial method,

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= - 1 + 13 - 32 + 20$

$= 33 - 33 = 0$

As $p(-1)$ is zero, therefore, $x + 1$ is a factor of this polynomial $p(x)$.

Let us find the quotient on dividing $x^3 + 13x^2 + 32x + 20$ by $(x + 1)$.

$$
\begin{array}{r l}
 & x^2 + 12x + 20 \\
\cline{2-2}
x+1 \big) & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & \quad 12x^2 + 32x \\
 & \quad 12x^2 + 12x \\
 & \quad - \quad - \\
\hline
 & \qquad 20x + 20 \\
 & \qquad 20x + 20 \\
 & \qquad - \quad - \\
\hline
 & \qquad\quad 0 \\
\hline
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder x^3 +

$13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20) + 0$

$= (x + 1)(x^2 + 10x + 2x + 20)$

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 10)(x + 2) =$

$(x + 1)(x + 2)(x + 10)$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$

By trial method, $p(1) = 2(1)^3 +$

$(1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1 = 0$

Therefore, $y - 1$ is a factor of this polynomial.

Let us find the quotient on dividing $2y^3 + y^2 - 2y - 1$ by $y - 1$.

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & \quad 3y^2 - 2y - 1 \\
 & \quad 3y^2 - 3y \\
 & \quad - \quad + \\
\hline
 & \qquad y - 1 \\
 & \qquad y - 1 \\
\hline
 & \qquad 0 \\
\hline
\end{array}
$$

$p(y) = 2y^3 + y^2 - 2y - 1 =$

$(y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)[2y(y + 1) + 1(y + 1)]$

$= (y - 1)(y + 1)(2y + 1)$ Exercise 2.5 Question 1:

Use suitable identities to find the following products:

(i) $(x+4)(x+10)$ (ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$ (iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Answer:

(i) By using the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$,

$$\begin{aligned}(x+4)(x+10) &= x^2 + (4+10)x + 4\times 10\\ &= x^2 + 14x + 40\end{aligned}$$

(ii) By using the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$,

$$\begin{aligned}(x+8)(x-10) &= x^2 + (8-10)x + (8)(-10)\\ &= x^2 - 2x - 80\end{aligned}$$

(iii) $(3x+4)(3x-5) = 9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)$

By using the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$,

$$\begin{aligned}9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right) &= 9\left[x^2 + \left(\frac{4}{3}-\frac{5}{3}\right)x + \left(\frac{4}{3}\right)\left(-\frac{5}{3}\right)\right]\\ &= 9\left[x^2 - \frac{1}{3}x - \frac{20}{9}\right]\\ &= 9x^2 - 3x - 20\end{aligned}$$

(iv) By using the identity $(x+y)(x-y) = x^2 - y^2$,

$$\begin{aligned}\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right) &= \left(y^2\right)^2 - \left(\frac{3}{2}\right)^2\\ &= y^4 - \frac{9}{4}\end{aligned}$$

(v) By using the identity $(x+y)(x-y) = x^2 - y^2$,

$$\begin{aligned}(3-2x)(3+2x) &= (3)^2 - (2x)^2\\ &= 9 - 4x^2\end{aligned}$$

Question 2:

Evaluate the following products without multiplying directly:

(i) 103 × 107 (ii) 95 × 96 (iii) 104 × 96 Answer: (i) 103 × 107 = (100 + 3) (100 + 7)

= $(100)^2$ + (3 + 7) 100 + (3) (7)

[By using the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$, where x =

100, a = 3, and b = 7]
= 10000 + 1000 + 21

= 11021

(ii) $95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 + (-5 - 4)100 + (-5)(-4)$

[By using the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$, where $x = 100$, $a = -5$, and $b = -4$]

$= 10000 - 900 + 20$

$= 9120$

(iii) $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$ $\left[(x+y)(x-y) = x^2 - y^2\right]$

$= 10000 - 16$

$= 9984$

Question 3:

Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Answer:

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)(3x + y) \qquad \left[x^2 + 2xy + y^2 = (x+y)^2\right]$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)(2y - 1) \qquad \left[x^2 - 2xy + y^2 = (x-y)^2\right]$

(iii) $x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2$

$= \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right) \qquad \left[x^2 - y^2 = (x+y)(x-y)\right]$

Question 4:

Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$ (ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$ (iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$ (vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Answer:

It is known that,

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

(i) $(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+2(x)(2y)+2(2y)(4z)+2(4z)(x)$

$= x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2 = (2x)^2+(-y)^2+(z)^2+2(2x)(-y)+2(-y)(z)+2(z)(2x)$

$= 4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

$= (-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(2z)(-2x)$
$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a-7b-c)^2$

$= (3a)^2+(-7b)^2+(-c)^2+2(3a)(-7b)+2(-7b)(-c)+2(-c)(3a)$
$= 9a^2+49b^2+c^2-42ab+14bc-6ac$

(v) $(-2x+5y-3z)^2$

$= (-2x)^2+(5y)^2+(-3z)^2+2(-2x)(5y)+2(5y)(-3z)+2(-3z)(-2x)$
$= 4x^2+25y^2+9z^2-20xy-30yz+12xz$

(vi) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$

$= \left(\frac{1}{4}a\right)^2+\left(-\frac{1}{2}b\right)^2+(1)^2+2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)+2\left(-\frac{1}{2}b\right)(1)+2\left(\frac{1}{4}a\right)(1)$
$= \frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$

Question 5:

Factorise:

$4x^2+9y^2+16z^2+12xy-24yz-16xz$ (i)

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$ (ii) Answer:

It is known that,

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(2x)(-4z)$$
$$= (2x+3y-4z)^2$$
$$= (2x+3y-4z)(2x+3y-4z)$$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$$= \left(-\sqrt{2}x\right)^2 + (y)^2 + \left(2\sqrt{2}z\right)^2 + 2\left(-\sqrt{2}x\right)(y) + 2(y)\left(2\sqrt{2}z\right) + 2\left(-\sqrt{2}x\right)\left(2\sqrt{2}z\right)$$
$$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$$
$$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)\left(-\sqrt{2}x + y + 2\sqrt{2}z\right)$$

Question 6:

Write the following cubes in expanded form:

(i) $(2x+1)^3$ (ii) $(2a-3b)^3$

(iii) $\left[\frac{3}{2}x+1\right]^3$ (iv) $\left[x-\frac{2}{3}y\right]^3$

Answer:

It is known that,

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$
$$\text{and } (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

(i) $(2x+1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x+1)$

$= 8x^3 + 1 + 6x(2x+1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a-3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$

$= 8a^3 - 27b^3 - 18ab(2a-3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $\left[\frac{3}{2}x+1\right]^3 = \left[\frac{3}{2}x\right]^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$

$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(vi) $\left[x-\frac{2}{3}y\right]^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

Question 7:

Evaluate the following using suitable identities:

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$ Answer:

It is known that,

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$
$$\text{and}\,(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

(i) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

= 1000000 − 1 − 300(99)

= 1000000 − 1 − 29700

= 970299

(ii) $(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

$= 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200 = 1061208$

(iii) $(998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$

$= 1000000000 - 8 - 6000(998)$

$= 1000000000 - 8 - 5988000$

$= 1000000000 - 5988008$

$= 994011992$ Question

8:

Factorise each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Answer:

It is known that,

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

and $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$$= (2a)^3 + (b)^3 + 3(2a)^2 b + 3(2a)(b)^2$$
$$= (2a+b)^3$$
$$= (2a+b)(2a+b)(2a+b)$$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$$= (2a)^3 - (b)^3 - 3(2a)^2 b + 3(2a)(b)^2$$
$$= (2a-b)^3$$
$$= (2a-b)(2a-b)(2a-b)$$

(iii) $27 - 125a^3 - 135a + 225a^2$

$$= (3)^3 - (5a)^3 - 3(3)^2(5a) + 3(3)(5a)^2$$
$$= (3-5a)^3$$
$$= (3-5a)(3-5a)(3-5a)$$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$$= (4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$$
$$= (4a-3b)^3$$
$$= (4a-3b)(4a-3b)(4a-3b)$$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2\left(\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2$$
$$= \left(3p - \frac{1}{6}\right)^3$$
$$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$$

Question 9:

Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Answer:

(i) It is known that,

$$(x+y)^3=x^3+y^3+3xy(x+y)$$

$$\begin{aligned} x^3+y^3 &= (x+y)^3-3xy(x+y) \\ &= (x+y)\left[(x+y)^2-3xy\right] \\ &= (x+y)(x^2+y^2+2xy-3xy) \\ &= (x+y)(x^2+y^2-xy) \\ &= (x+y)(x^2-xy+y^2) \end{aligned}$$

(ii) It is known that,

$$(x-y)^3=x^3-y^3-3xy(x-y)$$

$$\begin{aligned} x^3-y^3 &= (x-y)^3+3xy(x-y) \\ &= (x-y)\left[(x-y)^2+3xy\right] \\ &= (x-y)(x^2+y^2-2xy+3xy) \\ &= (x-y)(x^2+y^2+xy) \\ &= (x-y)(x^2+xy+y^2) \end{aligned}$$

Question 10:

Factorise each of the following:

(i) $27y^3+125z^3$

(ii) $64m^3-343n^3$

[Hint: See question 9.]

Answer:

(i) $27y^3 + 125z^3$

$$= (3y)^3 + (5z)^3$$
$$= (3y + 5z)\left[(3y)^2 + (5z)^2 - (3y)(5z)\right] \qquad \left[\because a^3 + b^3 = (a+b)(a^2 + b^2 - ab)\right]$$
$$= (3y + 5z)\left[9y^2 + 25z^2 - 15yz\right]$$

(ii) $64m^3 - 343n^3$

$$= (4m)^3 - (7n)^3$$
$$= (4m - 7n)\left[(4m)^2 + (7n)^2 + (4m)(7n)\right] \qquad \left[\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right]$$
$$= (4m - 7n)\left[16m^2 + 49n^2 + 28mn\right]$$

Question 11:

Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Answer:

It is known that,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$\therefore 27x^3 + y^3 + z^3 - 9xyz$$
$$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$$
$$= (3x + y + z)\left[(3x)^2 + y^2 + z^2 - (3x)(y) - (y)(z) - z(3x)\right]$$
$$= (3x + y + z)\left[9x^2 + y^2 + z^2 - 3xy - yz - 3xz\right]$$

Question 12:

Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$

Answer:

It is known that,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$=\frac{1}{2}(x+y+z)\left[2x^2+2y^2+2z^2-2xy-2yz-2zx\right]$$
$$=\frac{1}{2}(x+y+z)\left[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2zx)\right]$$
$$=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

Question 13:

If x + y + z = 0, show that $x^3+y^3+z^3-3xyz$.

Answer:

It is known that,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

Put x + y + z = 0,

$$x^3+y^3+z^3-3xyz=(0)(x^2+y^2+z^2-xy-yz-zx)$$
$$x^3+y^3+z^3-3xyz=0$$
$$x^3+y^3+z^3=3xyz$$

Question 14:

Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Answer:

(i) $(-12)^3+(7)^3+(5)^3$

Let x = −12, y = 7, and z = 5

It can be observed that, x + y
+ z = − 12 + 7 + 5 = 0

It is known that if x + y + z = 0, then

$x^3 + y^3 + z^3 = 3xyz$

$\therefore$ $(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$

= −1260

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let x = 28, y = −15, and z = −13

It can be observed that,

x + y + z = 28 + (−15) + (−13) = 28 − 28 = 0

It is known that if x + y + z = 0, then

$x^3 + y^3 + z^3 = 3xyz$

$$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$$
$$= 16380$$

Question 15:

Give possible expressions for the length and breadth of each of thefollowing rectangles, in which their areas are given:

$\boxed{\text{Area: } 25a^2 - 35a + 12}$	$\boxed{\text{Area: } 35y^2 + 13y - 12}$
I	II

Answer:

Area = Length × Breadth

The expression given for the area of the rectangle has to be factorised. One of its factors will be its length and the other will be its breadth.

(i) $25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12$

$= 5a(5a-3) - 4(5a-3)$

$= (5a-3)(5a-4)$

Therefore, possible length = 5a − 3

And, possible breadth = 5a − 4

(ii) $35y^2+13y-12=35y^2+28y-15y-12$

$=7y(5y+4)-3(5y+4)$

$=(5y+4)(7y-3)$

Therefore, possible length = 5y + 4

And, possible breadth = 7y − 3 Question 16:

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

I	II
Volume: $3x^2-12x$	Volume: $12ky^2+8ky-20k$

Answer:

Volume of cuboid = Length × Breadth × Height

The expression given for the volume of the cuboid has to be factorised. One of its factors will be its length, one will be its breadth, and one will be its height.

(i) $3x^2-12x=3x(x-4)$

One of the possible solutions is as follows.

Length = 3, Breadth = x, Height = x − 4

(ii) $12ky^2+8ky-20k=4k\left(3y^2+2y-5\right)$

$=4k\left[3y^2+5y-3y-5\right]$

$=4k\left[y(3y+5)-1(3y+5)\right]$

$=4k(3y+5)(y-1)$

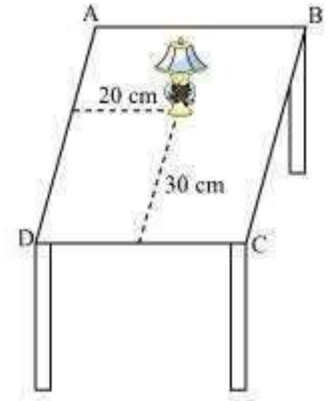
One of the possible solutions is as follows.

Length = 4k, Breadth = 3y + 5, Height = y − 1

Question 1:
Exercise 3.1

How will you describe the position of a table lamp on your study table to another person?

Answer:



Consider that the lamp is placed on the table. Choose two adjacent edges, DC and AD. Then, draw perpendiculars on the edges DC and AD from the position of lamp and measure the lengths of these perpendiculars. Let the length of these perpendiculars be 30 cm and 20 cm respectively. Now, the position of the lamp from the left edge (AD) is 20 cm and from the lower edge (DC) is 30 cm. This can also be written as (20, 30), where 20 represents the perpendicular distance of the lamp from edge AD and 30 represents the perpendicular distance of the lamp from edge DC.

Question 2:

(Street Plan): A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West direction.

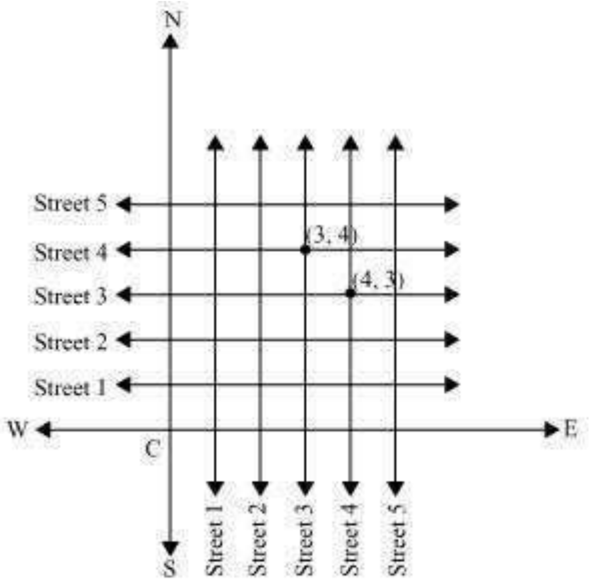
All the other streets of the city run parallel to these roads and are 200 m apart. There are about 5 streets in each direction. Using 1 cm = 100 m, draw a model of the city on your notebook Represent the roads/streets by single lines.

There are many cross-streets in your model. A particular cross-street is made by two streets, one running in the North-South direction and another in the East-West direction. Each cross street is referred to in the following manner: If the 2^{nd} street running in the North-South direction and 5^{th} in the East-West direction meet at some crossing, then we will call this cross-street (2, 5). Using this convention, find:

(i) How many cross - streets can be referred to as (4, 3).

(ii) How many cross - streets can be referred to as (3, 4).

Answer:



Both the cross-streets are marked in the above figure. It can be observed that there is only one cross-street which can be referred as (4, 3), and again, only one which can be referred as (3, 4).

Question 1:

Exercise 3.2

Write the answer of each of the following questions:

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

(ii) What is the name of each part of the plane formed by these two lines?

(iii) Write the name of the point where these two lines intersect.

Answer:

(i) The name of horizontal lines and vertical lines drawn to determine the position of any point in the Cartesian plane is x-axis and y-axis respectively.

(ii) The name of each part of the plane formed by these two lines, x-axis and y-axis, is quadrant (one-fourth part).

(iii) The name of the point where these two lines intersect is the origin.

Question 2:

See the given figure, and write the following:

(i) The coordinates of B.

(ii) The coordinates of C.

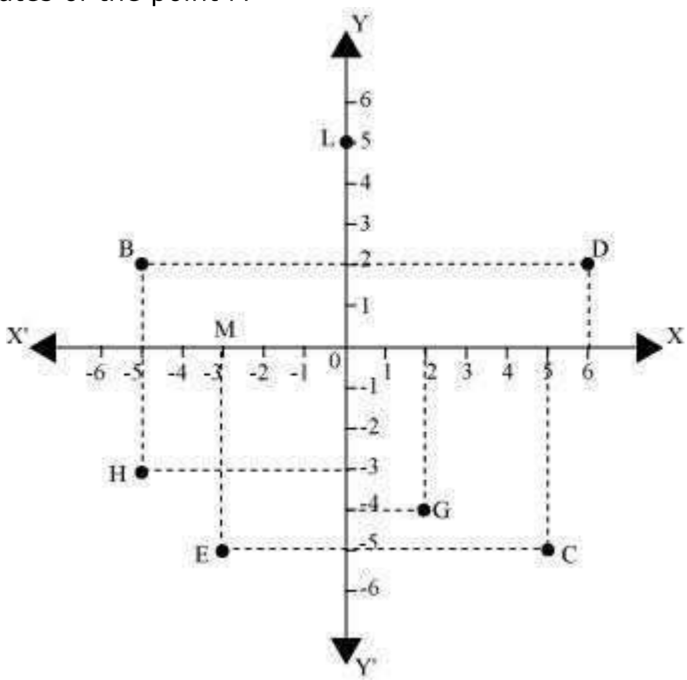
(iii) The point identified by the $(-3,-5)$ coordinates.

(iv) The point identified by the $(2,-4)$ coordinates (v) The abscissa of the point D.

(vi) The ordinate of the point H.

(vii) The coordinates of the point L.

(viii) The coordinates of the point M



Answer:

(i) The x-coordinate and the y-coordinate of point B are −5 and 2 respectively. Therefore, the coordinates of point B are (−5, 2).

(ii) The x-coordinate and the y-coordinate of point C are 5 and −5 respectively. Therefore, the coordinates of point C are (5, −5).

(iii) The point whose x-coordinate and y-coordinate are −3 and −5 respectively is point E.

Question 1:

(iv) The point whose x-coordinate and y-coordinate are 2 and −4 respectively is point G.

(v) The x-coordinate of point D is 6. Therefore, the abscissa of point D is 6.

(vi) The y-coordinate of point H is −3. Therefore, the ordinate of point H is −3.

(vii) The x-coordinate and the y-coordinate of point L are 0 and 5 respectively. Therefore, the coordinates of point L are (0, 5).

(viii) The x-coordinate and the y-coordinate of point M are −3 and 0 respectively. Therefore, the coordinates of point M is (−3, 0).

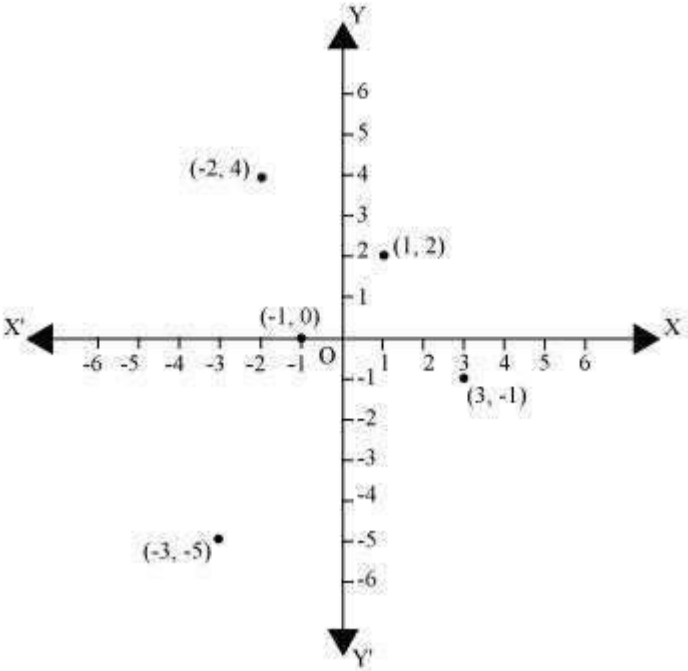
Exercise 3.3

$$(-2,4),(3,-1),(-1,0),(1,2)$$

In which quadrant or on which axis do each of the points

and $(-3,-5)$ lie? Verify your answer by locating them on the Cartesian plane.

Answer:



The point $(-2,4)$ lies in the IInd quadrant in the Cartesian plane because for point $(-2,4)$, x-coordinate is negative and y-coordinate is positive.

Again, the point $(3,-1)$ lies in the IV th quadrant in the Cartesian plane because for point $(3,-1)$, x-coordinate is positive and y-coordinate is negative.

$(-1,0)$ The point lies on negative x-axis because for point $(-1,0)$, the value of ycoordinate is zero and the value of x-coordinate is negative.

Question 1:

The point $(1,2)$ lies in the I^{st} quadrant as for point $(1,2)$, both x and y are positive.

The point $(-3,-5)$ lies in the III^{rd} quadrant in the Cartesian plane because for point $(-3,-5)$, both x and y are negative.

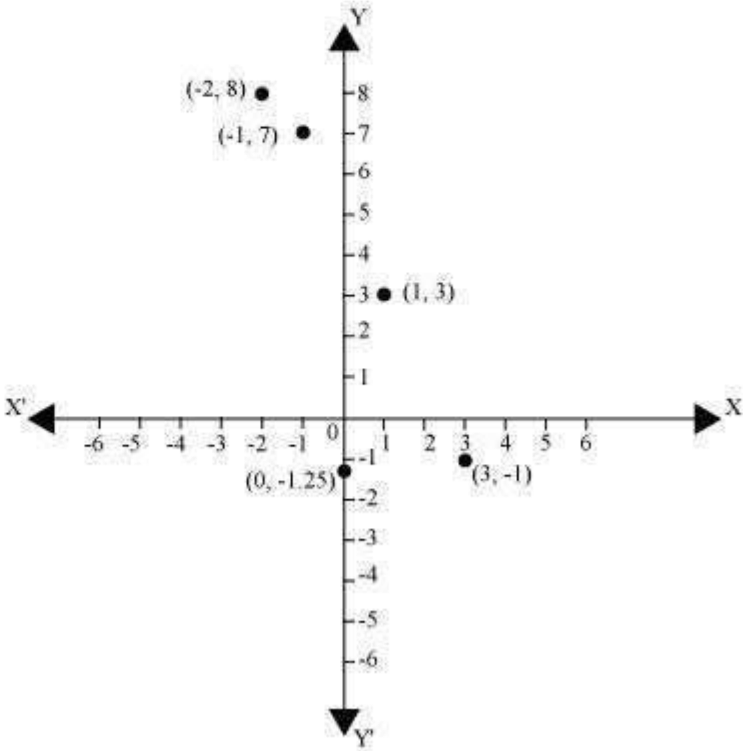
Question 2:

Plot the point (x, y) given in the following table on the plane, choosing suitable units of distance on the axis.

x	− 2	− 1	0	1	3
y	8	7	1.25	3	− 1

Answer:

The given points can be plotted on the Cartesian plane as follows.


Y
(-2, 8)
(-1, 7)
8
7
6
5
4
3
(1, 3)
2
1
X'
X
-6 -5 -4 -3 -2 -1
0
1 2 3 4 5 6
-1
(0, -1.25)
(3, -1)
-2
-3
-4
-5
-6
Y'